Properties of Solid

Unit: 4.1

Deforming force:

When an external force is applied on a body, which is not free to move, the shape and size of the body change. The force applied is called deforming force.

Restoring force:

When the deforming forces applied to a body are removed, the body tends to regain its original shape and size due to a force developed within the body. The force developed within the body, which is equal and opposite to deforming force is called restoring force.

Elasticity:

The property by virtue of which a body tends to regain its original shape and size after removal of the deforming force is known as elasticity.

Elastic body:

Bodies, which completely regain their original size and shape after the removal of the deforming force, are called elastic bodies.

Plastic body:

Bodies which change the shape and size on the application of force and which do not regain their original condition on removal of the deforming forces are said to be plastic bodies.

Stress:

When an external force is acting on an elastic body, it causes deformation (change in shape or in size or both). At the same time, a restoring force is developed within the material due to its elastic property. This force brings the body to its original shape and size.

The stress is defined as the restoring force acting on unit area.

Stress = Force/Area

Since the applied force and the restoring force are equal in magnitude, the stress is measured as the applied force acting per unit area.

SI unit for stress is **Nm^{-2}** or **pascal** (Pa)

Stress is of two types:

- *Normal stress:* It is defined as restoring force per unit area perpendicular to the surface of the body. Normal stress is of two types: *tensile stress* and *compressive stress*.
- *Tangential stress*: When the elastic deforming force or restoring force acts parallel to the surface, then the stress is called tangential stress.

Strain:

It is defined as the ratio of change in size or shape to the original size or shape. It has no dimensions. It is just a number.

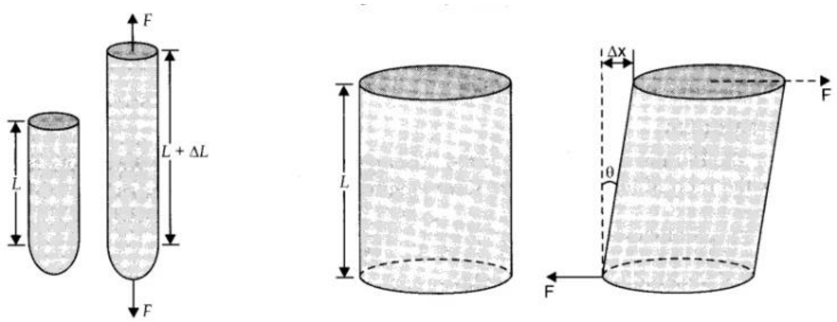

Fig. 4.1.1

Strain is of three types:

- **Longitudinal strain**: The longitudinal strain is defined as the ratio of change in length to the original length. As the linear strain is ratio of lengths, it has no unit.

  If the change (increase or decrease) in length is ΔL in a wire of original length L, then the longitudinal strain is given as

  Longitudinal strain = $\dfrac{\textbf{change in length } (\Delta \mathbf{L})}{\textbf{original length } (\mathbf{L})}$

- **Bulk (or) Volume strain:** Volume strain is defined as the ratio of change in volume to the original volume. It has also no unit.

  If ΔV is the change in volume produced in a body of original volume V, then volume strain is given as

  Volume strain = $\dfrac{\textbf{change in volume } (\Delta \mathbf{V})}{\textbf{original volume } (\mathbf{V})}$

- **Shearing (or) Rigidity strain:** When a force is applied parallel to one face of a body, the opposite side being fixed, there is a change in shape but not in size of the body. This strain is called the shearing strain. Solids alone can have a shearing strain. It is measured by the angle of the shear θ in radian.

Hooke's law:

Hooke's law states that *within the elastic limit, the ratio of the stress to the corresponding strain produced is a constant.* This constant is called *modulus of elasticity.* Thus,

Modulus of elasticity = $\frac{\textbf{stress}}{\textbf{strain}}$

Since strain is a pure number, the unit of this ratio is same as stress, i.e., Nm^{-2}.

Stress-strain curve:

Stress-strain curve are useful to understand the tensile strength of a given material. The given figure shows a stress-strain curve of a given metal.

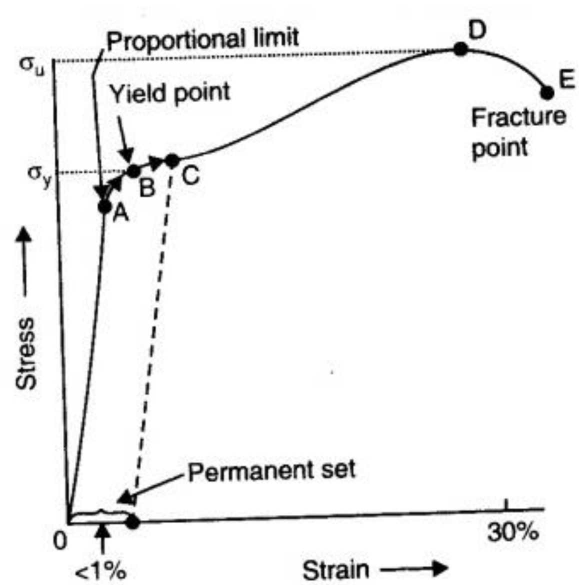

Fig. 4.1.2

- The curve from O to A is linear. In this region Hooke's proportional limit is obeyed.

- In the region from A to C, the stress and strain are not proportional; still the body regains its original dimension once the load is removed.

- Point B on the curve is the yield point or elastic limit and the corresponding stress is known as the yield strength of the material.

- The curve beyond B is known as the region of plastic deformation.

- The point D on the curve shows the tensile strength of the material. Beyond this point additional strain leads to fracture in the given material.

Moduli of Elasticity:

There are three types of moduli depending upon the three kinds of strain.

Young's modulus (Y):

Young's modulus is defined as the ratio of the longitudinal stress to the longitudinal strain.

Young's modulus, Y = $\frac{\textbf{longitudinal stress}}{\textbf{longitudinal strain}}$

Let a wire of initial length *L* and cross-sectional area *A* undergoes an extension *l* when a stretching force *F* is applied in the direction of its length. Then

longitudinal or linear stress = $\frac{F}{A}$

longitudinal strain = $\frac{\Delta L}{L}$

$$\boxed{\mathrm{Y} = \frac{\mathrm{F/A}}{\Delta\mathrm{L/L}} = \frac{\mathrm{F.L}}{\mathrm{A}\Delta\mathrm{L}}}$$

SI unit for Young's modulus is $\mathbf{Nm^{-2}}$ or **pascal** (Pa).

Bulk (or) Volume modulus ($\boldsymbol{\kappa}$):

When a body is subjected to a uniform compressive force, its volume decreases and the strain produced is a bulk or volume strain. *Bulk modulus is defined as the ratio of bulk stress to bulk strain.*

Bulk modulus, $\boldsymbol{\kappa}$ = $\frac{\textbf{Bulk stress}}{\textbf{Bulk strain}}$

If *F* is the total compressive force acting on a total area *A*, then

Bulk stress = *F/A* = *P*

If ΔV is the change in volume and *V* is the original volume, then

Bulk strain = $\frac{-\Delta V}{V}$

The negative sign indicates that increase in pressure causes decrease in volume.

$$\boxed{\kappa = \frac{P}{-\Delta V/V} = -\frac{PV}{\Delta V}}$$

SI unit for bulk modulus is **Nm⁻²** or **pascal** (Pa).

Shearing (or) Rigidity modulus (η):

The ratio of the shearing stress applied to the body to the shearing strain produced is called the rigidity modulus.

Rigidity modulus, $\boldsymbol{\eta = \frac{\text{shearing stress}}{\text{shearing strain}}}$

If T is the tangential force/unit area and if θ is the angle of shear measured in radian, then

$$\boxed{\eta = \frac{T}{\theta} = \frac{F}{A\theta}}$$

SI unit for rigidity modulus is $\mathbf{Nm^{-2}\ rad^{-1}}$ or $\mathbf{Pa\ rad^{-1}}$.

Poisson's ratio (σ):

When a tensile stress is applied to a wire, the wire undergoes not only an extension of length in the direction of the force but also a contraction in its thickness. The ratio of decrease in thickness to the original thickness in lateral direction is known as *lateral contraction*.

The ratio of lateral contraction to linear elongation is called Poisson's ratio.

Poisson's ratio, $\boldsymbol{\sigma = \frac{\text{lateral contraction}}{\text{linear elongation}}}$

Q.1 A wire of 2 m length and cross-sectional area 2 mm^2 elongates by 1 mm when a load of 15 kg is applied to it. Find the Young's modulus of the material of the wire.

Q.2 A wire of length 1m is stretched by a force of 10 N. The area of cross-section of the wire is is $2x10^{-6}$ m^2 and its Young's modulus is $2x10^{11}$ Nm^{-2}. Calculate (i) stress (ii) strain (iii) the increase in length of the wire.

Q.3 Calculate the force required to double the length of a steel wire of cross-sectional area 0.5 cm^2. The Young's modulus for steel is $2x10^{11}$ Nm^{-2}